ANALYSIS OF FAST REACTORS SYSTEMS

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INTRODUCTION

Fast reactors differ from thermal reactors in several aspects and require a special treatment.

The parasitic capture cross sections in the fuel, coolant and structure fall with increasing neutron energy faster than the fission cross section. Thus the neutron economy is improved in the fast region of the neutron spectrum. This is achieved by avoiding the use of moderator materials as coolants.

Since the fission cross section is less in a fast reactor than in a thermal reactor, a fast reactor would contain much more fissile fuel than a thermal reactor for the attainment of a critical mass.

Fast reactor cores, having no moderator, will be very compact in size. This leads to a higher power density necessitating the use of efficient coolants such as liquid metals. Sodium, lead, and a sodium-potassium eutectic that is liquid at room temperature are prime candidates.

A small core would imply a relatively large amount of neutron leakage from its surface. A reflector is avoided and is replaced by a blanket to intercept the leaking neutrons into a breeding material. The multiplication and energy production in the blanket must be accounted for from the perspective of power production.

We proceed to treat such a system using a subdivision into a fast group for neutrons of energy above 1.35 MeV, the fission threshold of U^{238}, and a thermal group for energies below it.

THE CORE AND BLANKET TWO GROUP EQUATIONS

The fast and thermal group core equations can be written as:

Core fast group:

$$\begin{aligned}&-[\text{Leakage from fast group}]-[\text{Absorptions in fast group}]\\&-[\text{Downscattering from fast to thermal group}]\\&+[\text{Fissions in fast group}]+[\text{Fissions in thermal group}]=0\\&+D_{1c}\nabla^2\phi_{1c}-\Sigma_{1ac}\,\phi_{1c}-\Sigma_{1s2c}\,\phi_{1c}+\chi_1\nu_{1c}\,\Sigma_{1fc}\,\phi_{1c}+\chi_1\nu_{2c}\,\Sigma_{2fc}\,\phi_{2c}=0\end{aligned} \tag{1}$$

Core thermal group:

-[Leakage from thermal group]-[Absorptions in thermal group]
+[Downscattering neutrons from fast group]
+[Fissions in fast group]+[Fissions in thermal group]=0 (2)

$$D_{2c}\nabla^2\phi_{2c} - \Sigma_{2ac}\,\phi_{2c} + \Sigma_{1s2c}\,\phi_{1c} + \chi_2\nu_{1c}\,\Sigma_{1fc}\,\phi_{1c} + \chi_2\nu_{2c}\,\Sigma_{2fc}\,\phi_{2c} = 0$$

We can notice that both fast and thermal neutrons are causing fissions in either group such as:

$$\chi_1 + \chi_2 = 1$$

The average numbers of neutrons per fission in different materials are listed in Table 1.

Table 1: Two group fast reactor constants. (From ANL-5800).
Group 1: Energy range 1.35 MeV to ∞, χ_1 = 0.575.
Group 2: Energy range 0 to 1.35 MeV, χ_2 = 0.425.

Element	ν		σ_f		σ_c		σ_{tr}		σ_{1s2}
	1	**2**	**1**	**2**	**1**	**2**	**1**	**2**	
Pu^{239}	3.10	2.93	1.95	1.78	0.10	0.30	4.6	7.0	0.90
U^{235}	2.70	2.50	1.29	1.44	0.08	0.28	4.5	7.2	1.50
U^{238}	2.60	2.47	0.524	0.005	0.036	0.19	4.6	7.1	2.05
Fe	-	-	-	-	0.005	0.006	2.0	2.8	0.70
Na	-	-	-	-	0.0005	0.0008	2.0	3.5	0.30
Al	-	-	-	-	0.004	0.002	1.8	3.5	0.38

As in the case of thermal reactors:

$$\Sigma_a = \Sigma_f + \Sigma_c .$$

In the absence of a moderator, the role of inelastic scattering is as important as the elastic scattering, so that:

$$\Sigma_{1s2} = \Sigma_{1in2} + \Sigma_{1e2} .$$

The diffusion coefficients are defined as:

$$D_1 = \frac{1}{3\Sigma_{1tr}}, D_2 = \frac{1}{3\Sigma_{2tr}},$$

where:

$$\Sigma_{1tr} = \Sigma_{1a} + \Sigma_{1in2} + \Sigma_{1e2} + \Sigma_{1in1} + \Sigma_{1e1}(1-\bar{\mu}_0)$$
$$\Sigma_{2tr} = \Sigma_{2a} + \Sigma_{2in2} + \Sigma_{2e2}(1-\bar{\mu}_0)$$

σ_{jej} is the elastic scattering cross section within group j

σ_{jinj} is the inelastic scattering cross section within group j

$\sum_{1tr}$, $\sum_{2tr}$ are the macroscopic transport cross sections, which are the reciprocals of the mean free paths for all interactions in groups 1 and 2.

In the same manner, we can now write the U^{238} blanket equations, neglecting the thermal fission of U^{238}.

Blanket fast group:

$$+D_{1b}\nabla^2\phi_{1b} - \Sigma_{1ab}\,\phi_{1b} - \Sigma_{1s2b}\,\phi_{1b} + \chi_1\nu_{1b}\,\Sigma_{1fb}\,\phi_{1b} = 0 \qquad (3)$$

Blanket thermal group:

$$+D_{2b}\nabla^2\phi_{2b} - \Sigma_{2ab}\,\phi_{2b} + \Sigma_{1s2b}\,\phi_{1b} + \chi_2\nu_{1b}\,\Sigma_{1fb}\,\phi_{1b} = 0 \qquad (4)$$

CORE AND BLANKET COUPLING COEFFICIENTS AND CRITICALITY DETERMINANT

If we let:

$$\nabla^2 = -B^2\phi\,,$$

we can rewrite Eqns. 1 and 2 in the form:

$$D_{1c}B^2\phi_{1c} + (\Sigma_{1ac} + \Sigma_{1s2c} - \chi_1\nu_{1c}\,\Sigma_{1fc})\phi_{1c} = \chi_1\nu_{2c}\,\Sigma_{2fc}\,\phi_{2c} \qquad (5)$$

$$D_{2c}B^2\phi_{2c} + (\Sigma_{2ac} - \chi_2\nu_{2c}\,\Sigma_{2fc})\phi_{2c} = (\Sigma_{1s2c} + \chi_2\nu_{1c}\,\Sigma_{1fc})\phi_{1c} \qquad (6)$$

Let us denote:

$$(\Sigma_{1ac} + \Sigma_{1s2c} - \chi_1\nu_{1c}\,\Sigma_{1fc}) = \Sigma_{1nc}$$
$$(\Sigma_{2ac} - \chi_2\nu_{2c}\,\Sigma_{2fc}) = \Sigma_{1p2c}$$

as the net core removal cross sections, and:

$$(\Sigma_{1s2c} + \chi_2\nu_{1c}\,\Sigma_{1fc}) = \Sigma_{1p2c}$$

as the total production cross section of group 2 neutrons from group 1 neutrons, and define:

$$L_{1nc}^2 = \frac{D_{1c}}{\sum_{1nc}}, L_{2nc}^2 = \frac{D_{2c}}{\sum_{2nc}}$$

then, Eqns. 5 and 6 become:

$$(1+\mathrm{L}_{1\mathrm{nc}}^2 B^2)\phi_{1\mathrm{c}} = \frac{\chi_1 \nu_{2c} \sum_{2\mathrm{fc}}}{\sum_{1nc}} \phi_{2\mathrm{c}} \tag{7}$$

$$(1+\mathrm{L}_{2\mathrm{nc}}^2 B^2)\phi_{2\mathrm{c}} = \frac{\sum_{\mathrm{ip2c}}}{\sum_{2nc}} \phi_{1\mathrm{c}} \tag{8}$$

Eliminating ϕ_{1c} and ϕ_{2c} from Eqns. 7 and 8, we get:

$$1 = k_n \frac{1}{(1+\mathrm{L}_{1\mathrm{nc}}^2 B^2)} \frac{1}{(1+\mathrm{L}_{2\mathrm{nc}}^2 B^2)} \tag{9}$$

where:

$$k_n = \frac{\chi_1 \nu_{2c} \sum_{2fc} \sum_{1p2c}}{\sum_{1nc} \sum_{2nc}} \tag{10}$$

having estimated the quantities L_{1nc}^2, L_{2nc}^2 and k_n, and similarly to the case of thermal reactors, we can estimate the principal and alternate bucklings as:

$$\mu^2 = \frac{-b + \sqrt{b^2 + 4c}}{2} \tag{11}$$

$$\nu^2 = \mu^2 + b \tag{12}$$

where:

$$b = \frac{1}{L_{1nc}^2} + \frac{1}{L_{2nc}^2}$$

$$c = \frac{k_n - 1}{L_{1nc}^2 . L_{2nc}^2}$$

For the principal buckling, $B^2 = \mu^2$,

$$\nabla^2 \phi_{1c} + \mu^2 \phi_{1c} = 0 \ , \ \phi_{1c} = AX$$
$$\nabla^2 \phi_{2c} + \mu^2 \phi_{2c} = 0 \ , \ \phi_{2c} = A' X$$

By substituting in Eqn. 2:

$$-\mathrm{D}_{2c}\mu^2 A^{'}\mathrm{X} - \Sigma_{2nc} A^{'}\mathrm{X} + \Sigma_{1p2c} A\mathrm{X} = 0\,,$$

from which we can get the principal coupling coefficient as:

$$S_1 = \frac{A^{'}}{A} = \frac{\Sigma_{1p2c}}{(\Sigma_{2nc} + D_{2c}\mu^2)} \tag{13}$$

For the alternate buckling, $B^2 = \nu^2$,

$$\nabla^2\phi_{1c} - \nu^2\phi_{1c} = 0 \;\;,\;\; \phi_{1c} = CY$$
$$\nabla^2\phi_{2c} - \nu^2\phi_{2c} = 0 \;\;,\;\; \phi_{2c} = C^{'}Y$$

By substituting in Eqn. 2,

$$S_2 = \frac{C^{'}}{C} = \frac{\Sigma_{1p2c}}{(\Sigma_{2nc} + D_{2c}\nu^2)} \tag{14}$$

To estimate the coupling coefficients in the reflector, we define the removal cross section in the blanket as:

$$\Sigma_{1nb} = \Sigma_{1ab} + \Sigma_{1s2b} - \chi_1\nu_{1b}\,\Sigma_{1fb}$$

and the total production cross section of thermal neutrons from fast group neutrons:

$$\Sigma_{1p2b} = \Sigma_{1s2b} + \chi_2\nu_{2b}\,\Sigma_{1fb}$$

We can thus write Eqns. 3 and 4 as:

$$\nabla^2\phi_{1b} - \frac{\Sigma_{1nb}}{D_{1b}}\phi_{1b} = 0 \tag{15}$$

$$\nabla^2\phi_{2b} - \frac{\Sigma_{2ab}}{D_{2b}}\phi_{2b} + \frac{\Sigma_{1p2b}}{D_{2b}}\phi_{1b} = 0 \tag{16}$$

Defining the diffusion areas:

$$L_{1b}^2 = \frac{D_{1b}}{\Sigma_{1nb}}, L_{2b}^2 = \frac{D_{2b}}{\Sigma_{2ab}}\,,$$

the last equations become:

$$\nabla^2 \phi_{1b} - \frac{1}{L_{1b}^2} \phi_{1b} = 0 \tag{17}$$

$$\nabla^2 \phi_{2b} - \frac{1}{L_{2b}^2} \phi_{2b} + \frac{\Sigma_{1p2b}}{D_{2b}} \phi_{1b} = 0 \tag{18}$$

In the same way as in the case of thermal reactors, we get:

$$S_3 = \frac{\Sigma_{1p2b}}{D_{2b}} \frac{1}{(\frac{1}{L_{2b}^2} + \frac{1}{L_{1b}^2})} \tag{19}$$

If the currents and fluxes continuity conditions are applied at the core and blanket boundary we obtain a 4 by 4 criticality determinant as in the case of thermal reactors, but with a different definition of the coupling coefficients and constants. If the reactor composition is varied, then a single value of the critical radius for the core can be obtained.

ESTIMATION OF THE FLUX DISTRIBUTIONS

We seek to obtain expressions for the flux distributions normalized to 1 Watt of reactor power production. The difference between fast reactors and thermal reactors is that fast fissions are generating power in the blanket. Thus the total fast reactor power can be written as:

$$\begin{aligned} P[\text{Watts}] &= (\Sigma_{2fc}\, \overline{\phi}_{2c} V_c + \Sigma_{1fc}\, \overline{\phi}_{1c} V_c + \Sigma_{1fb}\, \overline{\phi}_{1b} V_b) \frac{\text{fissions}}{\text{cm}^3.\text{sec}}.\ \text{cm}^3.200 \frac{\text{MeV}}{\text{fission}} 1.6x10^{-13} \frac{\text{Joule}}{\text{MeV}} \\ &= 3.2x10^{-11} (\Sigma_{2fc}\, \overline{\phi}_{2c} V_c + \Sigma_{1fc}\, \overline{\phi}_{1c} V_c + \Sigma_{1fb}\, \overline{\phi}_{1b} V_b) \end{aligned} \tag{20}$$

We define the fast fission factor as:

$$\begin{aligned} \varepsilon &= \frac{\text{Total fission rate in core and blanket}}{\text{Thermal group energy fissions in core}} \\ &= 1 + \frac{\Sigma_{1fc}\, \overline{\phi}_{1c}}{\Sigma_{2fc}\, \overline{\phi}_{2c}} + \frac{\Sigma_{1fb}\, \overline{\phi}_{1b} V_b}{\Sigma_{2fc}\, \overline{\phi}_{2c} V_c} \end{aligned} \tag{21}$$

From Eqns. 20 and 21, we get:

$$P[\text{Watts}] = 3.2x10^{-11} \varepsilon\, \Sigma_{2fc}\, \overline{\phi}_{2c} V_c \tag{22}$$

To compute the power P, one needs to estimate the fast fission factor ε. This in turn needs the knowledge of:

$$\bar{\phi}_{1c}, \bar{\phi}_{2c}, \bar{\phi}_{1b} .$$

The first two are the same as the fast and thermal core fluxes $\bar{\phi}_{1c}, \bar{\phi}_{2c}$ in the case of thermal reactors associated with the appropriate constants.

The quantity $\bar{\phi}_{1b}$ can be written as:

$$\bar{\phi}_{1b} = \frac{\int_V \phi_{1b} dV_b}{\int_V dV_b} = \frac{\int_R^{R+T} \frac{F}{r} \sinh \frac{(R+T+d-r)}{L_{1b}} 4\pi r^2 dr}{\int_R^{R+T} 4\pi r^2 dr}$$

$$= \frac{3F}{(R+T)^3 - R^3} \int_R^{R+T} r \sinh \frac{(R+T+d-r)}{L_{1b}} dr$$

where the integration is carried out over the blanket thickness.

Now:

$$\begin{aligned}
I &= \int_a^{b-d} r \sinh\left(\frac{b-r}{c}\right) - c \int_a^{b-d} rd[\cosh\left(\frac{b-r}{c}\right)] \\
&= -c[r\cosh\left(\frac{b-r}{c}\right) - c\int_a^{b-d} \cosh\left(\frac{b-r}{c}\right) dr]_a^{b-d} \\
&= -c[r\cosh\left(\frac{b-r}{c}\right) + c\sinh\left(\frac{b-r}{c}\right)]_a^{b-d} \\
&= -c\left(b-d\right)\cosh\left(\frac{d}{c}\right) + a\cosh\left(\frac{b-a}{c}\right) - c^2\sinh\left(\frac{d}{c}\right) + c^2\sinh\left(\frac{b-a}{c}\right)
\end{aligned}$$

Thus:

$$\begin{aligned}
\bar{\phi}_{1b} = \frac{3F}{(R+T)^3 - R^3} &[RL_{1b}\cosh\left(\frac{T+d}{L_{1b}}\right) + L_{1b}^2 \sinh\left(\frac{T+d}{L_{1b}}\right) \\
&- L_{1b}(R+d)\cosh\left(\frac{d}{L_{1b}}\right) - L_{1b}^2\sinh\left(\frac{d}{L_{1b}}\right)]
\end{aligned} \tag{23}$$

Thus the fast fission factor ε can be evaluated from Eqn. 21. The quantity A can be calculated for P = 1 Watt, and the fluxes can be plotted.

ESTIMATION OF THE BREEDER RATIO

In fast reactor systems based upon the U^{235} and Pu^{239} fuel cycle, it is possible to produce more fissile fuel atoms than those that are consumed. As a measure of this capability we define the "breeding ratio" BR as:

$$\text{BR}=\frac{\text{Rate of fissile atoms production}}{\text{Rate of fissile atoms consumption}} \tag{24}$$

When BR<1, it is denoted as the "conversion ratio" instead.

If α is the capture to fission ratio, then the number of fissile atoms consumed per second in the core:

$$(1+\alpha_1)\sum_{1fc}\bar{\phi}_{1c}V_c+(1+\alpha_2)\sum_{2fc}\bar{\phi}_{2c}V_c$$

Writing down the number of fissile atoms production in the core and blanket, Eqn. 24 can be written as:

$$\begin{aligned} BR &= \frac{\sum_{1cc}\bar{\phi}_{1c}V_c+\sum_{2cc}\bar{\phi}_{2c}V_c+\sum_{1cb}\bar{\phi}_{1b}V_b+\sum_{2cb}\bar{\phi}_{2b}V_b}{(1+\alpha_1)\sum_{1fc}\bar{\phi}_{1c}V_c+(1+\alpha_2)\sum_{2fc}\bar{\phi}_{2c}V} \\ &= \frac{\sum_{1cc}\dfrac{\bar{\phi}_{1c}}{\bar{\phi}_{2c}}+\sum_{2cc}+(\sum_{1cb}\bar{\phi}_{1b}+\sum_{2cb}\bar{\phi}_{2b})\dfrac{V_b}{V_c\bar{\phi}_{2c}}}{(1+\alpha_1)\sum_{1fc}\dfrac{\bar{\phi}_{1c}}{\bar{\phi}_{2c}}+(1+\alpha_2)\sum_{2fc}} \end{aligned} \tag{25}$$

where $\sum_{1c},\sum_{2c}$ are the capture cross sections in the core for groups 1 and 2.

In Eqn. 25 we need to find an expression for $\bar{\phi}_{2b}$. This can be done in the following way:

$$\begin{aligned} \bar{\phi}_{2b} &= \frac{\int_V \phi_{2b}dV_b}{\int_V dV_b}=\frac{\int_V (S_3\phi_1+GZ_2)dV_b}{V_b} \\ &= S_3\bar{\phi}_{1b}+\frac{4\pi G}{V_b}\int_R^{R+T} r\sinh\frac{(R+T+d-r)}{L_{2b}}dr \end{aligned}$$

Making use of the result in Eqn. 23, we can write:

$$\overline{\phi}_{2b} = S_3\overline{\phi}_{1b} + \frac{3G}{(R+T)^3 - R^3}[RL_{2b}\cosh\left(\frac{T+d}{L_{2b}}\right) + L_{2b}^2\sinh\left(\frac{T+d}{L_{2b}}\right)$$
$$- L_{2b}(R+d)\cosh\left(\frac{d}{L_{2b}}\right) - L_{2b}^2\sinh\left(\frac{d}{L_{2b}}\right)] \quad (26)$$

Knowing $\overline{\phi}_{2b}$ from eqn. 26 and $\overline{\phi}_{1b}$ from Eqn. 23, the value of the breeding ration can be determined from Eqn. 25.

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